

Calculators and mobile phones are not allowed  
Answer all of the questions

1. (a) Let  $f(x) = \tan^{-1}(\ln x)$ . Prove that  $f$  is one-to-one and find its inverse. State the domain and range of  $f^{-1}$ . (5 points)

(b) Let  $f(x) = 2^{3x} + 2 \cdot 5^x - 5$ . Find an equation of the tangent line to the graph of  $f^{-1}$  at the point  $P(-2, 0)$ . (3 points)

2. Find  $\frac{dy}{dx}$ , if

(a)

$$y = \frac{e^{\operatorname{sech} x} \sqrt{x^2 - 2}}{\ln x (\sin^{-1} x)^x}$$

(b)

$$\log_3 |e + y| - 3^{-y} = 6.$$

(4+3 points)

3. Verify the following identities:

(a)

$$\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \tan^{-1}(x), \quad \text{for } x \geq 0.$$

(b)

$$\cosh(\ln x) - \sinh(\ln x) = \frac{1}{x}, \quad \text{for } x > 0.$$

(3+3 points)

4. Evaluate

(a)

$$\lim_{x \rightarrow \infty} x[\ln(x-1) - \ln(x)]$$

(b)

$$\lim_{x \rightarrow \infty} \left(\cos \frac{1}{x}\right)^x$$

(4+3 points)

5. Evaluate the following integrals

(a)

$$\int_0^1 \frac{x}{\sqrt{4-x^4}} dx$$

(b)

$$\int \frac{(e^{-x} + e^{2x})^2}{e^{3x}} dx$$

(c)

$$\int \frac{\sec(\ln x)}{x} dx$$

(d)

$$\int \frac{1}{e^{1-x} + 1} dx$$

(3 points each)

Total 40 points